DIFFERENTIATION FORMULAS

Some Useful Rules	
Product Rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
Division Rule $v \neq 0$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$
Chain Rule	$\frac{d}{dx}(f(g(x))) = f'(g(x)) \, . \, g'(x)$
Addition/ Subtraction	$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$
Constant Rule	$\frac{d}{dx}(k\, f(x)) = k\frac{d}{dx}(f(x))$

Some Standard Derivatives	
$\frac{d}{dx}(\text{constant}) = 0$	$\frac{d}{dx}(x^n) = nx^{n-1}$
$\frac{d}{dx}(\log_e x) = \frac{1}{x}$	$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(\log_a x) = \frac{1}{x\log_e a}$	$\frac{d}{dx}(a^x) = a^x \log_e a$

$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\sin x) = \cos x$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$
$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$	$\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$
$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{\lvert x \rvert \sqrt{x^2-1}}$	$\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{\lvert x \rvert \sqrt{x^2-1}}$
$\frac{d}{dx}(\sinh x) = \cosh x$	$\frac{d}{dx}(\cosh x) = \sinh x$
$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$	$\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$
$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$	
$\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$	

$\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}}$	$\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}}$
$\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$	$\frac{d}{dx}(\coth^{-1}x) = \frac{1}{x^2-1}$
$\frac{d}{dx}(\text{sech}^{-1}x) = -\frac{1}{\lvert x\rvert\sqrt{1-x^2}}$	$\frac{d}{dx}(\text{cosech}^{-1}x) = \frac{-1}{\lvert x\rvert\sqrt{x^2+1}}$
$\frac{d}{dx}\lvert x\rvert = \frac{x}{\lvert x\rvert}\ (x \neq 0)$	$\frac{d}{dx}[x] = 0, \forall x \in -I$
$\frac{d}{dx}\log\lvert x\rvert = \frac{1}{x}$	$\frac{d}{dx}\{x\} = 1, \forall x \in R$

Some Important Substitutions

$\sqrt{a^2-x^2}$	• $x = a\sin\theta$ or $a\cos\theta$
$\sqrt{x^2+a^2}$	• $x = a\tan\theta$ or $a\cot\theta$
$\sqrt{x^2-a^2}$	• $x = a\sec\theta$ or $a\,\text{cosec}\theta$
$\sqrt{\frac{a-x}{a+x}}$	• $x = a\cos 2\theta$
$\sqrt{(x-a)(x-b)}$	• $x = a\sec^2\theta - b\tan^2\theta$
$\sqrt{(x-a)(b-x)}$	• $x = a\cos^2\theta + b\sin^2\theta$
$\sqrt{ax-x^2}$	• $x = a\sin^2\theta$

DIFFERENTIATION OF IMPLICIT FUNCTION

- When in a function the dependent variable is not explicitly isolated on either side of the equation

Working Method

- Every term of f(x, y) = 0 should be differentiated with respect to x
- The value of dy/dx should be obtained by rearranging the terms.

EXAMPLE

$$x^4 + y^3 - 3x^2y = 0$$

$$4x^3 + 3y^2\frac{dy}{dx} - 3\left(2xy + x^2.\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{4x^3 - 6xy}{3x^2 - 3y^2}$$

DIFFERENTIATION OF PARAMETRIC FORM

if x = f(t) and y = g(t), then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Example, $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1+\cos\theta)} = \tan\frac{\theta}{2}$$